ESTIMATING THE TECHNOLOGY OF CHILDREN'S SKILL FORMATION

Francesco Agostinelli and Matthew Wiswall

University of Pennsylvania

University of Wisconsin-Madison

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Presentation:
黄涛
曹诗韵 and 乔纪
杨雅欣 and 曹韵诗
王祎
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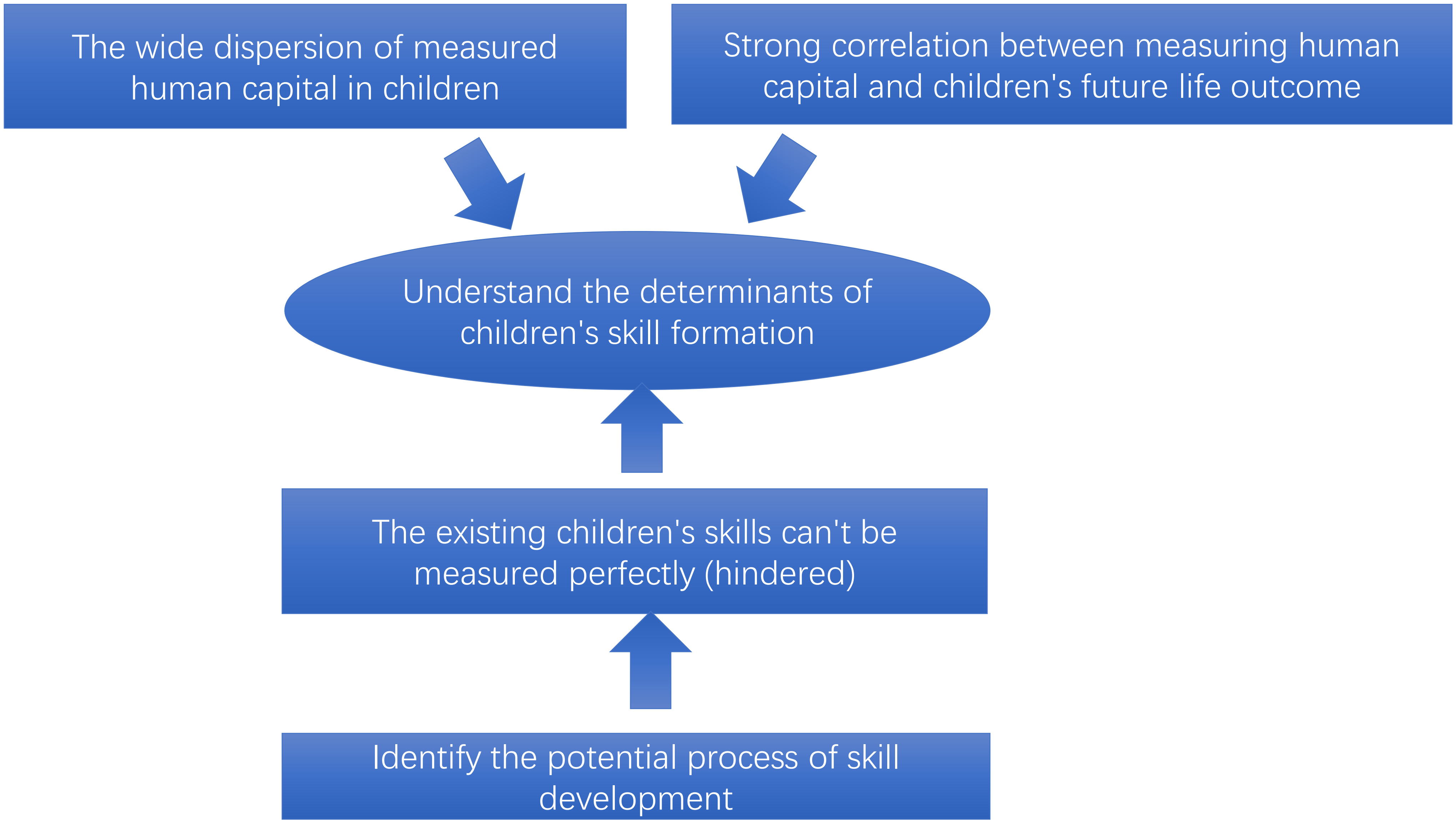
1 Introduction



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1 Introduction

This paper makes two contributions.

An alternative scheme has been developed.

- Use estimation framework
- Experience-based skill measurement limitations
- the specific parameter categories of the model are considered in the analysis.

A skill formation model is estimated.

- There was an interaction between children's skill growth at that time and parents' investment ability.
- It can estimate the heterogeneity of parents' investment returns and the complementarity between early and late investments at different stages of children's development.

1 Introduction

contribution:

1. The concept of "age invariant" measurement is analyzed

- Compare the skills development of growing children
- Implies limitations on how measurements are related during development.
- These assumptions do not apply to all measurements
- Indicate: if this type of measures are available, more classes can be determined to form technology; Not available, consider alternative restrictions
- There is a trade-off between the measurement model and the limitations of skill and technology

2. Provide some guidance for empirical researchers

- Age-standardized measurement (Z-score) cannot guarantee the comparability needed to identify potential skills and technologies , Age-invariant measurements require assumptions about the relationship between unobserved skills and observed measurements

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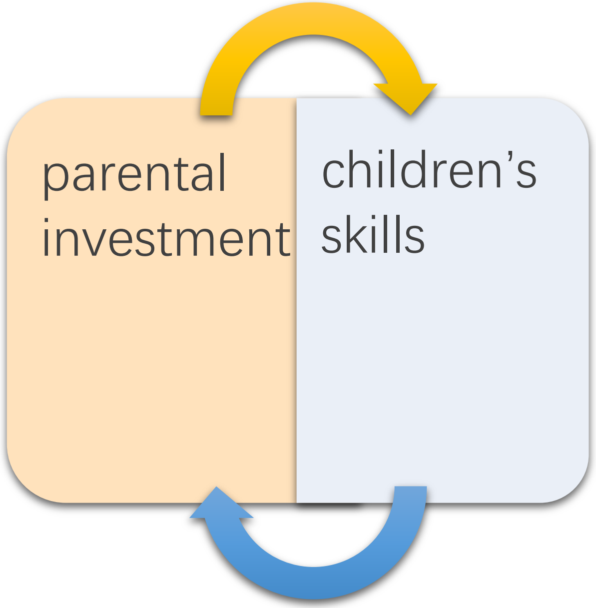
1 Introduction

Using data from the US **National Longitudinal Survey of Youth (NLSY)**:

This paper studies the development of cognitive skills of children aged 5-14, and designs a skill development model considering the complementarity between parents' investment and children's skills.



The **influencing factors** of children's skills: parents' investment, mothers' skills, family income, Hicks neutral dynamics in total factor productivity and free scale income.

Following Cunha et al. (2010), our empirical framework treats not only the child's cognitive skills as measured with error, but investment and maternal skills as well.

1 Introduction

a multiple step instrumental variable estimator.

The measured values in NLSY children's cognitive achievement data set (PIAT score) indicate the development and change of children's skills.

Finding:

- The best target of intervention measures is vulnerable children
- The impact on low-income families is greater

When comparing the above estimation with the estimation using the model that ignores the measurement error, the former has less policy effect, which shows that the method we adopt is quantitatively important for answering key policy questions.

Next——

Three parts: empirical model, identification analysis and estimation procedure using simplified model.

The rest: empirical results, comparison with existing estimates, and policy counterfactual results.

2 Stylized Model

- Skill Production Technology

$$\theta_{i,t+1} = h_t(\theta_{i,t}, I_{i,t}, \eta_{i,\theta,t}) \text{ for } t = 0, 1, \ldots, T-1 \tag{1}$$

  - $\theta_{i,t}$: skill stock
  - $I_{i,t}$: flow investment
  - $\eta_{i,\theta,t}$: production shock
  - $h_t(\cdot)$: production technology can vary as children age

$$\ln \theta_{t+1} = \ln A_t + \psi_t \ln f_t(\theta_t, I_t) + \eta_{\theta,t} \qquad \text{for } t = 0, 1, \ldots, T-1\,, \tag{2}$$

  - $f_t(\theta_t, I_t)$
    - location: the total factor productivity (TFP) term $\ln A_t$
    - scale: the return to scale parameter ψ_t
    - E.g. a constant return to scale Cobb-Douglas function, a more general CES function, or various “trans-log” functions

2 Stylized Model

- Policy-Relevant Effects of Interest
  - The features of the technology of skill development inform **the optimal timing of policy interventions** – the optimal investment portfolio across early and late childhood and **the optimal targeting of policy** – to which children should scarce resources be allocated to.
    - The productivity of investments at various child ages
    - How does heterogeneity in children's skills affect the productivity of new investments in children?
    - How do investments in children persist over time and affect adult outcomes?
      - Do early investments have a high return because they increase the productivity of later investments (dynamic complementarities) or do early investments "fade-out" over time?
  - Goal: using childhood interventions to affect eventual adult outcomes

2 Stylized Model

- Complementarities and Heterogeneity in Skill Production
  - The previous empirical work (Cunha and Heckman, 2008 ; Cunha et al., 2010)
    - parametric specifications for the f_t function assuming CES forms
    - the marginal return to parental investments is assumed to be (weakly) positive with respect to the current stock of skills

$$\frac{\partial^2 \theta_{t+1}}{\partial I_t \partial \theta_t} \geq 0 \; \forall t.$$

    - the heterogeneous marginal products: the marginal product of parental investments is larger for higher skilled children
  - **The empirical model**

$$\ln \theta_{t+1} = \ln A_t + \gamma_{1,t} \ln \theta_t + \gamma_{2,t} \ln I_t + \gamma_{3,t} \ln I_t \cdot \ln \theta_t + \eta_{\theta,t}, \quad (3)$$

    - $\gamma_{3,t}$: a free parameter that characterizes the heterogeneity in the returns to parental investments
      - ➢ positive value: higher productivity for highly skilled children
      - ➢ negative value: higher productivity for skill disadvantaged children

3 Measurement and Identification

- Measurement Model
  - Children's skills are measured by **multiple** measures, which can have some relationship to the unobserved latent skill stock θ_t.
  - For each period t, we have M_t measures for latent skills $\ln \theta_t$.

$$Z_{t,m} = \mu_{t,m} + \lambda_{t,m} \ln \theta_t + \epsilon_{t,m} \qquad \text{for } t = 0, 1, \ldots, T \text{ and } m = 1, \ldots, M_t. \tag{4}$$

    - $\mu_{t,m}$: the location of the measures
    - $\lambda_{t,m}$: the scale of the measures
    - $\epsilon_{t,m}$: the measurement error, $E(\epsilon_{t,m}) = 0$
  - Assumptions
    - Measurement errors are independent of latent skills.
    - Investment and independent of each other at each period t and investments are observed.

3 Measurement and Identification

- Under-Identification Problem
  - Identify the distribution of initial latent skills (t = 0)
    - normalization in the initial period and at least 3 measures of skills in this period
  - **Cannot separately identify** the location and scale of the measures in periods after the initial one from the scale and location of the production technology.

$$
\begin{aligned}
E(Z_{t+1,m}) - E(Z_{t,m}) &= (\mu_{t+1,m} - \mu_{t,m}) + \lambda_{t+1,m} E(\ln\theta_{t+1}) - \lambda_{t,m} E(\ln\theta_t) \\
&= \underbrace{(\mu_{t+1,m} - \mu_{t,m}) + (\lambda_{t+1,m} - \lambda_{t,m})}_{\text{measurement}} E(\ln\theta_t) + \lambda_{t+1,m} \underbrace{(E(\ln\theta_{t+1}) - E(\ln\theta_t))}_{\text{latent skills}}
\end{aligned}
$$

  - **Cannot infer** changes in the productivity of investments as children age from changes in the marginal product of mean measures.

$$
\begin{aligned}
\frac{\partial E(Z_{t+1,m}|I_t)}{\partial I_t} - \frac{\partial E(Z_{t,m}|I_{t-1})}{\partial I_{t-1}} &= \lambda_{t+1,m} \frac{\partial E(\ln\theta_{t+1}|I_t)}{\partial I_t} - \lambda_{t,m} \frac{\partial E(\ln\theta_t|I_{t-1})}{\partial I_{t-1}} \\
&= \underbrace{(\lambda_{t+1,m} - \lambda_{t,m})}_{\text{measurement}} \frac{\partial E(\ln\theta_t|I_{t-1})}{\partial I_{t-1}} + \lambda_{t+1,m} \underbrace{\left(\frac{\partial E(\ln\theta_{t+1}|I_t)}{\partial I_t} - \frac{\partial E(\ln\theta_t|I_{t-1})}{\partial I_{t-1}}\right)}_{\text{latent productivity}}
\end{aligned}
$$

    - **Without further restrictions**, we cannot simply use changes in the sensitivity of measures to investment to infer "sensitive" or "critical" periods.

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3 Measurement and Identification

Under-Identification Problem

$$\ln \theta_{t+1} = \ln A_t + \psi_t \ln f_t(\theta_t, I_t) + \eta_{\theta,t} \qquad \text{for } t = 0, 1, \dots, T-1\,, \qquad (2)$$

- The technology (2) has a free location and scale: ln A_t parameter (TFP in levels) and the ψ_t parameter (returns to scale in levels).

$$Z_{t,m} = \mu_{t,m} + \lambda_{t,m} \ln \theta_t + \epsilon_{t,m} \qquad \text{for } t = 0, 1, \dots, T \text{ and } m = 1, \dots, M_t. \qquad (4)$$

$$Z_{t+1,m} = \underbrace{(\mu_{t+1,m} + \lambda_{t+1,m} \ln A_t)}_{=\beta_{0,t,m}} + \underbrace{(\lambda_{t+1,m}\psi_t)}_{=\beta_{1,t,m}} \ln f_t(\theta_t, I_t) + u_{t,m}, \qquad (5)$$

- The technology location and scale parameters (A_t, ψ_t) are **not separately identified from** the next period measurement location and scale parameters ($\mu_{t+1,m}$, $\lambda_{t+1,m}$).

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3 Measurement and Identification

- Age-Invariance
  - Definition of age-invariant measures

    Definition 1 *A pair of measures* $Z_{t,m}$ *and* $Z_{t+1,m}$ *is* age-invariant *if* $E(Z_{t,m}|\theta_t = p) = E(Z_{t+1,m}|\theta_{t+1} = p)$ *for all* $p \in \mathbb{R}_{++}$.

    - Two children with the same level of latent skill would on average perform equally well, independently of their age.
  - Age-invariance is a kind of **factor model restriction**.
    - The only relevant variable for the measure is the level of latent skill possessed by the child, not the child's age directly.
    - The measurement parameters for a specific age-invariant measure m are constant over the two age periods($\mu_{t,m} = \mu_{t+1,m}$ and $\lambda_{t,m} = \lambda_{t+1,m}$).
  - Whether a given pair of measures is age-invariant **depends on** the measures available, and must be evaluated on a case-by-case basis.
    - Certain test score measures developed specifically to track development as children age are age-invariant. Examples of these types of measures for the cognitive skill domain include the Peabody Individual Achievement Test (PIAT) and the Woodcock-Johnson tests.
    - In the **empirical application**, we use the **PIAT measures** and discuss in more detail why we believe these measures are age-invariant.

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3 Measurement and Identification

- Cobb-Douglas Example
  - Skills developed in period 1 is specified as

$$\ln \theta_1 = \ln A_0 + \psi_0(\gamma_0 \ln \theta_0 + (1-\gamma_0)\ln I_0) + \eta_{\theta,0} \quad (6)$$

    - $\gamma_0 \in (0, 1)$
    - $\ln A_0$ and ψ_0 represent the location and scale of the production
    - Assume that the initial period (t = 0) measurement parameters $\{\mu_{0,m}, \lambda_{0,m}\}$m are already identified.
    - "error-contaminated" measures

$$\widetilde{Z}_{0,m} \equiv \frac{Z_{0,m} - \mu_{0,m}}{\lambda_{0,m}} = \ln \theta_0 + \widetilde{\epsilon}_{0,m} \,,$$

  - The next period measure is a function of latent skills as follows

$$Z_{1,m} = \mu_{1,m} + \lambda_{1,m} \ln \theta_1 + \epsilon_{1,m}$$

$$\begin{aligned} Z_{1,m} &= \mu_{1,m} + \lambda_{1,m}\ln A_0 + \lambda_{1,m}\psi_0(\gamma_0 \ln\theta_0 + (1-\gamma_0)\ln I_0) + \lambda_{1,m}\eta_{\theta,0} + \epsilon_{1,m} \\ &= (\mu_{1,m} + \lambda_{1,m}\ln A_0) + \lambda_{1,m}\psi_0(\gamma_0(\widetilde{Z}_{0,m} - \widetilde{\epsilon}_{0,m}) + (1-\gamma_0)\ln I_0) + \lambda_{1,m}\eta_{\theta,0} + \epsilon_{1,m} \\ &= \beta_{0,0} + \beta_{0,1}\widetilde{Z}_{0,m} + \beta_{0,2}\ln I_0 + \pi_{0,m} \end{aligned} \quad (7)$$

$$\pi_{0,m} = \lambda_{1,m}\eta_{\theta,0} + \epsilon_{1,m} - \lambda_{1,m}\psi_0\gamma_0\widetilde{\epsilon}_{0,m}$$

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3 Measurement and Identification

- **Cobb-Douglas Example**
  - Under-identification problem: **5 unknown** primitive parameters ($\ln A_0$, ψ_0, γ_0, $\mu_{0,m}$, $\lambda_{1,m}$) and only **3 identified** reduced form parameters (β_s)

$$\beta_{0,0} = \mu_{1,m} + \lambda_{1,m} \ln A_0,$$

$$\beta_{0,1} = \lambda_{1,m}\psi_0\gamma_0,$$

$$\beta_{0,2} = \lambda_{1,m}\psi_0(1-\gamma_0)$$

  - Assume that the measures Z0,m, Z1,m constitute a pair of **age-invariant measures** (Definition 1).
    - This implies the measurement restriction $\mu_{0,m} = \mu_{1,m} = \mu_m$ and $\lambda_{0,m} = \lambda_{1,m} = \lambda_m$.
  - Given the identification of the initial period measurement parameters $\mu_{0,m}$, $\lambda_{0,m}$

$$\psi_0 = \frac{\beta_{0,1}+\beta_{0,2}}{\lambda_m},$$

$$\gamma_0 = \frac{\beta_{0,1}}{\beta_{0,1}+\beta_{0,2}},$$

$$\ln A_0 = \frac{\beta_{0,0}-\mu_m}{\lambda_m}.$$

4 .Estimation

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4.1 Functional Forms

1.Multidimensional Initial Conditions

$$\Omega = (\ln \theta_0, \ln \theta_{MC}, \ln \theta_{MN}, \ln Y_0)$$

for counterfactual exercises ——estimate the initial relationship between latent skills and family income

$\Omega \sim N(\mu_\Omega, \Sigma_\Omega)$ where $\mu_\Omega = [0, 0, 0, \mu_{0,\ln Y}]$

2.Parental Investments

$$\ln I_t = \alpha_{1,t} \ln \theta_t + \alpha_{2,t} \ln \theta_{MC} + \alpha_{3,t} \ln \theta_{MN} + \alpha_{4,t} \ln Y_t + \eta_{I,t}$$

$\eta_{I,t}$:investment shock, iid ~ $N(0, \sigma^2_{I,t})$ for all t; independent of latent skills and income

(lnYt) follows an AR(1) process:

$$\ln Y_{t+1} = \mu_Y + \rho_Y \ln Y_t + \eta_{Y,t}$$

3.Skill Technolog——translog form

$$\ln \theta_{t+1} = \ln A_t + \gamma_{1,t} \ln \theta_t + \gamma_{2,t} \ln I_t + \gamma_{3,t} \ln I_t \cdot \ln \theta_t + \eta_{\theta,t},$$

production shock $\eta_{\theta,t} \text{i.i.d.} \sim N(0, \sigma^2_{\theta,t})$. independent of θ_t, I_t

4 Adult Outcome

adult outcomes:years of schooling measured at age 23 and log earnings at age 29.

$$Q = \mu_Q + \alpha_Q \ln \theta_T + \eta_Q,$$

η_Q Independent of $\ln \theta_T$

5.Measurement

$$Z_{I,t,m} = \mu_{I,t,m} + \lambda_{I,t,m} \ln I_t + \epsilon_{I,t,m}$$ for all t,m

$$Z_{\theta,t,m} = \mu_{\theta,t,m} + \lambda_{\theta,t,m} \ln \theta_t + \epsilon_{\theta,t,m}$$ for all t,m

$$Z_{MC,m} = \mu_{MC,m} + \lambda_{MC,m} \ln \theta_{MC} + \epsilon_{MC,m}$$ for all m

$$Z_{MN,m} = \mu_{MN,m} + \lambda_{MN,m} \ln \theta_{MN} + \epsilon_{MN,m}$$ for all m

Errors are independent of each other contemporaneously,independent of each other over-time,independent of latent variables, and independent of production and investment shocks

4.2 Estimation Algorith

——advantage/disadvantage of the sequential algorithm

- estimator does not require the simulation of thefull model;
- a joint estimation approach is by breaking the estimator into steps, which makes the identification assumptions as transparent as possible;
- loss of efficiency

Step 0 (Estimate Initial Conditions and Initial Measurement Parameters) Pre:杨雅欣 and曹韵诗

——Estimate the measurement parameters at the initial period (age 5-6)

$$\lambda_{\theta,0,m} = \frac{Cov(Z_{\theta,0,m}, Z_{\theta,0,m'})}{Cov(Z_{\theta,0,1}, Z_{\theta,0,m'})} \forall m \neq m',$$

$$\lambda_{\omega,m} = \frac{Cov(Z_{\omega,m}, Z_{\omega,n'})}{Cov(Z_{\omega,1}, Z_{\omega,m'})} \forall m \neq m' \text{and} \forall \omega \in \{MC, MN\}.$$

——Initial measurement intercepts

$$\mu_{\theta,0,m} = E(Z_{\theta,0,m}) \forall m$$

$$\mu_{\omega,m} = E(Z_{\omega,m}) \forall m \text{and} \forall \omega \in \{MC, MN\}.$$

——Form the following "residual" measure

$$\widetilde{Z}_{\theta,0,m} = \frac{Z_{\theta,0,m} - \mu_{\theta,0,m}}{\lambda_{\theta,0,m}} \forall m$$

$$\widetilde{Z}_{\omega,m} = \frac{Z_{\omega,m} - \mu_{\omega,m}}{\lambda_{\omega,m}} \forall m \text{and} \forall \omega \in \{MC, MN\}.$$

Step 1 (Estimate Investment Function Parameters)

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$$\frac{Z_{I,0,m} - \mu_{I,0,m} - \epsilon_{I,0,m}}{\lambda_{I,0,m}} = \alpha_{1,0}(\widetilde{Z}_{\theta,0,m} - \widetilde{\epsilon}_{\theta,0,m}) + \alpha_{2,0}(\widetilde{Z}_{MC,m} - \widetilde{\epsilon}_{MC,m}) + \alpha_{3,0}(\widetilde{Z}_{MN,m} - \widetilde{\epsilon}_{MN,m}) + \alpha_{4,0} \ln Y_0 + \eta_{I,0}$$

Re-arranging：

$$\begin{aligned} Z_{I,0,m} &= \mu_{I,0,m} + \lambda_{I,0,m}\alpha_{1,0}\widetilde{Z}_{\theta,0,m} + \lambda_{I,0,m}\alpha_{2,0}\widetilde{Z}_{MC,m} + \lambda_{I,0,m}\alpha_{3,0}\widetilde{Z}_{MN,m} + \lambda_{I,0,m}\alpha_{4,0} \ln Y_0 \\ &+ \epsilon_{I,0,m} + \lambda_{I,0,m}(\eta_{I,0} - \widetilde{\epsilon}_{\theta,0,m} - \widetilde{\epsilon}_{MC,m} - \widetilde{\epsilon}_{MN,m}) \\ &= \beta_{0,0,m} + \beta_{1,0,m}\widetilde{Z}_{\theta,0,m} + \beta_{2,0,m}\widetilde{Z}_{MC,m} + \beta_{3,0,m}\widetilde{Z}_{MN,m} + \beta_{4,0,m} \ln Y_0 + \pi_{I,0,m} \qquad (11) \end{aligned}$$

where $\widetilde{\epsilon}_{\theta,0,m} = \frac{\epsilon_{\theta,0,m}}{\lambda_{\theta,0,m}}$, $\widetilde{\epsilon}_{MC,m} = \frac{\epsilon_{MC,m}}{\lambda_{MC,m}}$, $\widetilde{\epsilon}_{MN,m} = \frac{\epsilon_{MN,m}}{\lambda_{MN,m}}$, $\beta_{j,0,m} = \lambda_{I,0,m}\alpha_{j,0}$ for all j
and

$$\pi_{I,0,m} = \epsilon_{I,0,m} + \lambda_{I,0,m}(\eta_{I,0} - \alpha_{1,0}\widetilde{\epsilon}_{\theta,0,m} - \alpha_{2,0}\widetilde{\epsilon}_{MC,m} - \alpha_{3,0}\widetilde{\epsilon}_{MN,m}).$$

Step 1 (Estimate Investment Function Parameters)

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Estimation of (11) by OLS would yield **inconsistent** estimates of the$\beta_{j,0,m}$coeffi-cients because the measures are correlated with their measurement errors (includedin the residual term$\pi_{I,0,m}$)

↓

Use instrumental variable estimator： $[Z_{\theta,0,m'}, Z_{MC,m'}, Z_{NC,m'}]$

↓

Obtain consistent estimators for the$\beta_{j,t,m}$ coefficients：

$$\alpha_{j,0} = \frac{\beta_{j,0,m}}{\sum_{j=1}^{4} \beta_{j,0,m}} \forall j \in \{1, \ldots, 4\}$$

Step 2 (Compute Measurement Parameters for Latent Investment):

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The intercept and factor loading for the investment measure are given by:

$$\mu_{I,0,m} = \beta_{0,0,m} \qquad \text{and} \qquad \lambda_{I,0,m} = \sum_{j=1}^{4} \beta_{j,0,m}$$

"residual" measures for investment in period $t = 0$:

$$\widetilde{Z}_{I,0,m} = \frac{Z_{I,0,m} - \mu_{I,0,m}}{\lambda_{I,0,m}} \equiv \ln I_0 + \widetilde{\epsilon}_{I,0,m}$$

Where $\widetilde{\epsilon}_{I,0,m} = \frac{\epsilon_{I,0,m}}{\lambda_{I,0,m}}$

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Step 3 (Estimate Skill Production Technology)

Substituting the residual measures into the production technology (3), we have:

$$\frac{Z_{\theta,1,m}-\mu_{\theta,1,m}-\epsilon_{\theta,1,m}}{\lambda_{\theta,1,m}} = \ln A_0 + \gamma_{1,0}(\widetilde{Z}_{\theta,0,m}-\widetilde{\epsilon}_{\theta,0,m}) + \gamma_{2,0}(\widetilde{Z}_{I,0,m}-\widetilde{\epsilon}_{I,0,m})$$
$$+\gamma_{3,0}(\widetilde{Z}_{\theta,0,m}-\widetilde{\epsilon}_{\theta,0,m})(\widetilde{Z}_{I,0,m}-\widetilde{\epsilon}_{I,0,m}) + \eta_{\theta,0}$$

$$Z_{\theta,1,m} = \delta_{0,0,m} + \delta_{1,0,m}\tilde{Z}_{\theta,0,m} + \delta_{2,0,m}\tilde{Z}_{I,0,m} + \delta_{3,0,m}\tilde{Z}_{\theta,0,m}\cdot\tilde{Z}_{I,0,m} + \pi_{\theta,0,m}, \quad (12)$$

where the new error term $\pi_{\theta,0,m}$ is:

$$\pi_{\theta,0,m} = \epsilon_{\theta,1,m} + \lambda_{\theta,1,m}[\eta_{\theta,0} - \gamma_{1,0}\widetilde{\epsilon}_{\theta,1,0,m} - \gamma_{2,0}\widetilde{\epsilon}_{1,0,m} - \gamma_{3,0}(\widetilde{Z}_{\theta,0,m}\widetilde{\epsilon}_{1,0,m} + \widetilde{Z}_{1,0,m}\widetilde{\epsilon}_{\theta,0,m} - \widetilde{\epsilon}_{\theta,0,m}\widetilde{\epsilon}_{I,0,m})].$$

Step 3 (Estimate Skill Production Technology)

The rest of the reduced-form parameters (δs) map into the structural parameters and measurement parameters in the following way:

$$\delta_{0,0,m} = \mu_{\theta,1,m} + \lambda_{\theta,1,m} \cdot \ln A_0$$

$$\delta_{j,0,m} = \lambda_{\theta,1,m}\gamma_{j,0} \text{for any} j \in \{1,2,3\}$$

Estimation of (12) using OLS would lead toan inconsistent estimator.

With IV： $[Z_{\theta,0,m'}, Z_{I,0,m'}, Z_{\theta,0,m'}, Z_{I,0,m'}]$,we can then recover the structural parameters:

$$\ln A_0 = \frac{\delta_{0,0,m} - \mu_{\theta,1,m}}{\lambda_{\theta,1,m}}$$

$$\gamma_{j,0} = \frac{\delta_{j,0,m}}{\lambda_{\theta,1,m}} \forall j \in \{1,2,3\}$$

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Step 4 (Estimate Variance of Investment and Production Function Shocks):

Use the covariance between the residual term ($\pi_{I,0,m}$) in (11), and an alternative residual measure of investment $\widetilde{Z}_{I,0,m'} = \ln I_0 + \widetilde{\epsilon}_{I,0,m'}$

$$Cov\left(\frac{\pi_{I,0,m}}{\lambda_{I,0,m}}, \widetilde{Z}_{I,0,m'}\right) = V(\eta_{I,0}) = \sigma^2_{I,0}.$$

To compute the residual measure $Z_{I,0,m}$:

$$Cov\left(\frac{\pi_{\theta,1,m}}{\lambda_{\theta,1,m}}, \widetilde{Z}_{\theta,1,m'}\right) = V(\eta_{\theta,0}) = \sigma^2_{\theta,0}$$

Remaining Steps :

Estimate equations for both years of education at age 23 and logearnings at age 29. With IV method:

$$Q = \mu_Q + \alpha_Q \widetilde{Z}_{\theta,4,m} + (\eta_Q - \alpha_Q \widetilde{\epsilon}_{\theta,4,m})$$

Standard Errors :

- To account for the sources of estimation uncertainty among different steps andthe sample design of the data ==》Bootstrap algorithm
- To account for the intra-family correlation ==》Block bootstrap algorithm

4.3 Data

- Source: National Longitudinal Survey of Youth 1979 (**NLSY79**)
- Match female with their children from Children and Young Adults surveys from 1986 to 2012
- Observations of age 5-6 through adulthood, 11,509 children
- Multiple measures
  - children's skills
    - Peabody Individual Achievement Test (**PIAT**) : Mathematics (age-invariant) , Reading and Recognition
    - Peabody Picture Vocabulary Test (**PPVT**)
  - mother's skills
    - mother's cognitive skills: Armed Services Vocational Aptitude Battery (**ASVAB**)
    - mother's non-cognitive skills: Rotter and Rosenberg indexes
  - parental investments
  - family income
  - children's highest grade completed at age 23 or older
  - children's earnings at age 29

Pre:王祎

Table B-8: Descriptive Statistics about Children's Cognitive Skills Measures

Measures	Mean	Std	Min	Max	Number of Values
Age 5-6					
PIAT Math	11.858	4.278	0.000	37.000	32.000
PIAT Recognition	12.864	5.048	0.000	57.000	35.000
PIAT Comprehensive	12.770	4.930	0.000	49.000	35.000
Age 7-8					
PIAT Math	23.016	8.681	0.000	74.000	58.000
PIAT Recognition	25.748	8.774	0.000	80.000	67.000
PIAT Comprehensive	24.099	8.142	0.000	69.000	60.000
Age 9-10					
PIAT Math	38.720	10.832	0.000	84.000	71.000
PIAT Recognition	40.825	11.487	0.000	84.000	76.000
PIAT Comprehensive	37.540	10.231	0.000	78.000	64.000
Age 11-12					
PIAT Math	48.184	10.543	0.000	84.000	78.000
PIAT Recognition	51.079	13.278	0.000	84.000	74.000
PIAT Comprehensive	45.732	11.272	0.000	84.000	72.000
Age 13-14					
PIAT Math	53.767	11.387	0.000	84.000	78.000
PIAT Recognition	58.670	14.262	0.000	84.000	74.000
PIAT Comprehensive	51.015	12.229	0.000	84.000	74.000

Table B-9: Descriptive Statistics of Parental Investment Measures

	Parental Investments			
Measures	Mean	Std	Min	Max
How often mom reads to child	4.22	1.41	1	6
How often mom eats with child	3.32	1.61	0	5
How often child was taken to museum	2.19	0.97	1	5
How often child is praised	5.56	4.37	0	20
How often complimented child	4.70	4.05	0	20

5. Results

- Parameter Estimates
- Estimated Child Development Path
- Policy Experiments
- Quantifying the Importance of Measurement Error
- Cost-Benefit Analysis

1. Parameter Estimates

Pre:王祎

Table B-2: Estimates for $\boxed{\text{Investment Equation}}$ — $\ln I_t = \alpha_{1,t} \ln \theta_t + \alpha_{2,t} \ln \theta_{MC} + \alpha_{3,t} \ln \theta_{MN} + \alpha_{4,t} \ln Y_t + \eta_{I,t}$

Model	Age 5-6	Age 7-8	Age 9-10	Age 11-12
Log Skills at age 5	0.230 *** (0.059) [0.15,0.33]	0.027 *** (0.009) [0.02,0.04]	0.020 ** (0.009) [0.01,0.04]	0.018 ** (0.009) [0.01,0.04]
Log Mother Cognitive Skills	0.071 *** (0.022) [0.04,0.12]	0.004 (0.009) [-0.01,0.02]	0.012 (0.015) [-0.01,0.04]	-0.005 (0.013) [-0.03,0.02]
Log Mother Noncognitive Skills	0.359 *** (0.130) [0.11,0.53]	0.742 *** (0.059) [0.64,0.82]	0.694 *** (0.083) [0.53,0.80]	0.712 *** (0.087) [0.54,0.81]
Log Family Income	0.341 *** (0.076) [0.25,0.48]	0.227 *** (0.056) [0.16,0.32]	0.274 *** (0.076) [0.17,0.42]	0.275 *** (0.087) [0.17,0.45]
Standard Deviation Shocks	1.186 (0.230) [0.97,1.54]	1.019 (0.147) [0.83,1.29]	0.868 (0.235) [0.66,1.32]	1.087 (0.295) [0.82,1.72]

1. Parameter Estimates

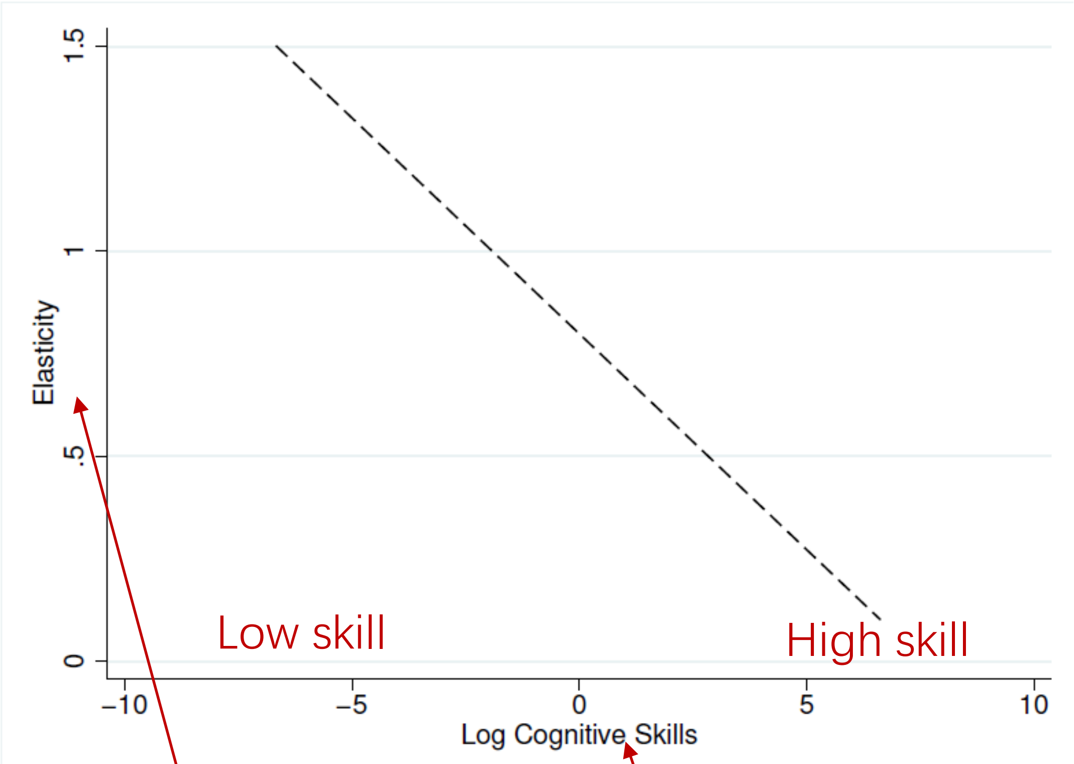
Pre:王祎

$$\ln\theta_{t+1} = \ln A_t + \gamma_{1,t}\ln\theta_t + \gamma_{2,t}\ln I_t + \gamma_{3,t}\ln I_t \cdot \ln\theta_t + \eta_{\theta,t}$$

Table 2: Estimates for the $\boxed{\text{Skill Technology}}$

Model	Age 5-6	Age 7-8	Age 9-10	Age 11-12
Log Skills	1.966 ***	1.086 ***	0.897 ***	1.065 ***
	(0.153)	(0.035)	(0.027)	(0.029)
	[1.71,2.21]	[1.03,1.15]	[0.85,0.93]	[1.02,1.11]
Log Investments	0.799 ***	0.695 **	0.713 *	0.252
	(0.261)	(0.337)	(0.403)	(0.538)
	[0.43,1.22]	[0.16,1.21]	[-0.03,1.24]	[-0.53,1.16]
Log Skills × Log Investments	-0.105 †	-0.005	-0.003	0.003
	(0.066)	(0.019)	(0.013)	(0.010)
	[-0.21,-0.03]	[-0.04,0.03]	[-0.02,0.02]	[-0.02,0.02]
Standard Deviation Shocks	5.612	4.519	3.585	4.019
	(0.173)	(0.184)	(0.180)	(0.246)
	[5.38,5.93]	[4.28,4.88]	[3.27,3.87]	[3.71,4.43]
Log TFP	13.067	14.747	11.881	2.927
	(0.294)	(0.365)	(0.538)	(0.952)
	[12.67,13.60]	[14.24,15.47]	[11.20,12.93]	[1.40,4.48]

Figure 1: Estimates of Skill Production Elasticity with Respect to Investment at Age 5-6



$$\frac{\partial\ln\theta_1}{\partial\ln I_0} = \gamma_{2,0} + \boldsymbol{\gamma_{3,0}}\ln\theta_0$$

1. Parameter Estimates

Pre:王祎

$Q = \mu_Q + \alpha_Q \ln \theta_T + \eta_Q$

Table B-3: Estimates for Adult Outcome Equation

Model	Schooling	Log Wage
Constant	7.088	8.394
	(0.397)	(0.252)
	[6.56,7.71]	[8.03,8.79]
Log Children Skills at age 13-14	0.151	0.041
	(0.009)	(0.006)
	[0.14,0.16]	[0.03,0.05]
Variance Shocks	4.333	0.875
	(0.142)	(0.064)
	[4.07,4.56]	[0.77,0.97]

1 unit ↑

Increase of 0.15 years of completed education at age 23
0.041 increase in log earnings at age 29

2. Estimated Child Development Path

Pre:王祎

Figure 3: Estimated Mean of Log Latent Skills

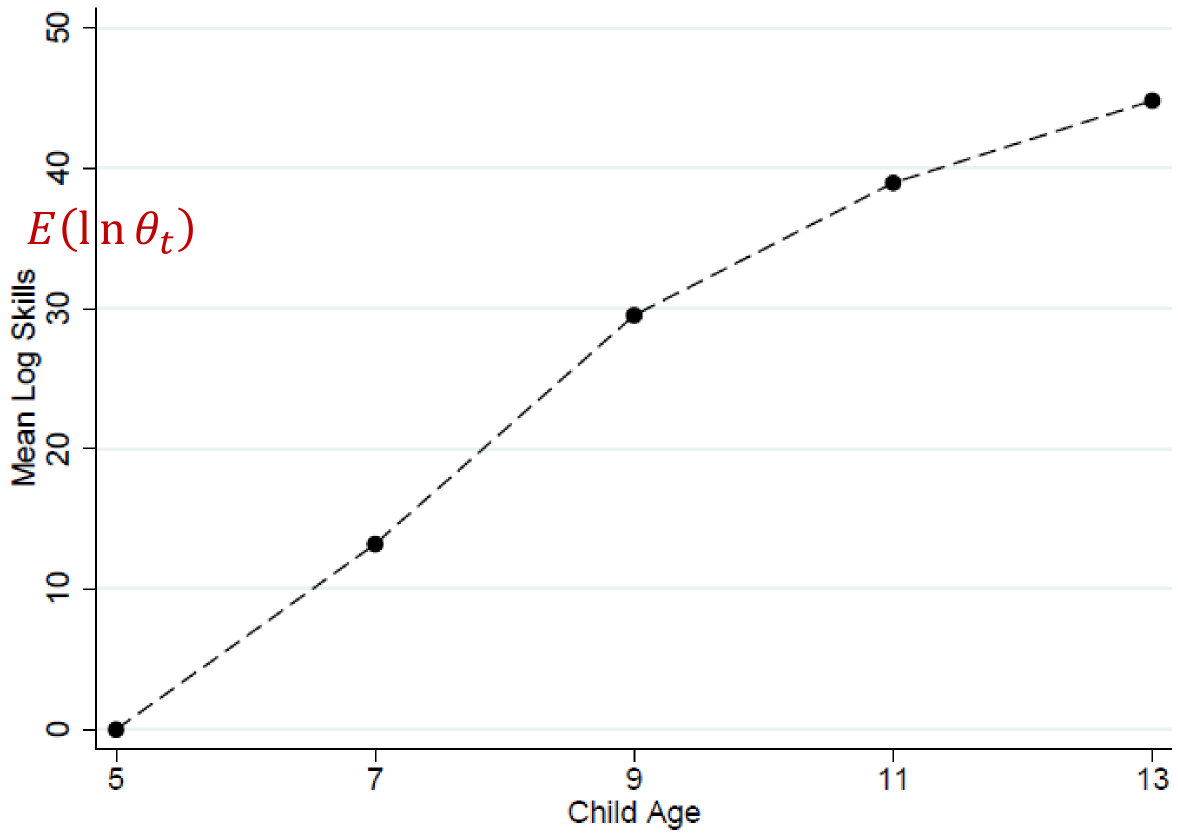
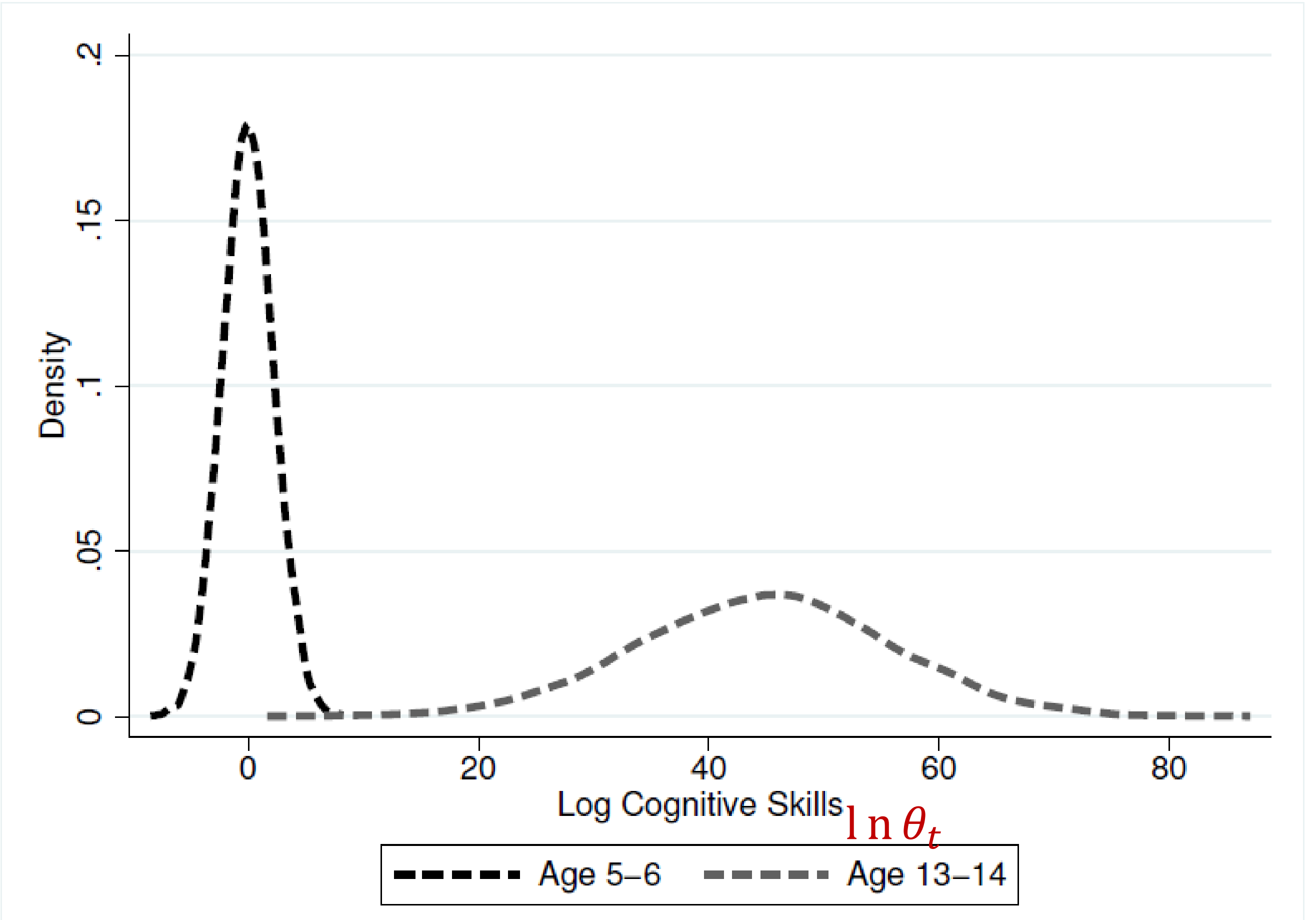


Figure 4: Estimated Distribution of Log Cognitive Latent Skills at Age 5-6 and Age 13-14



3. Policy Experiments

Pre:王祎

- Analyze the effect of income transfers on childhood skill development and adult outcomes
- The transfer is \$10,000 in family income at age t=a.

Figure 7: Average Effect of Income Transfer by Age of Transfer (Outcome: Final Period $\ln\theta_T$ Skills)

$E[\ln\theta_T'(a) - \ln\theta_T]$: at age 13-14

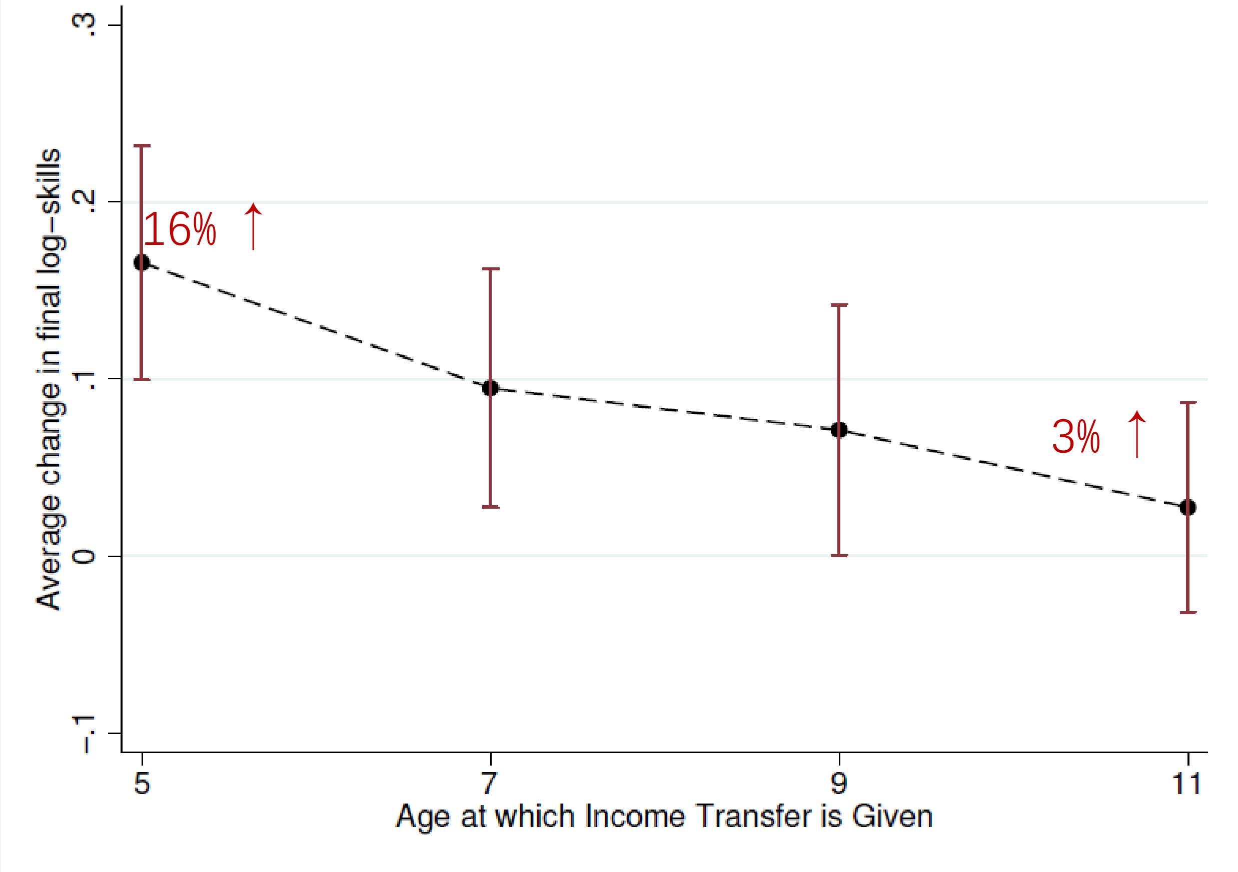
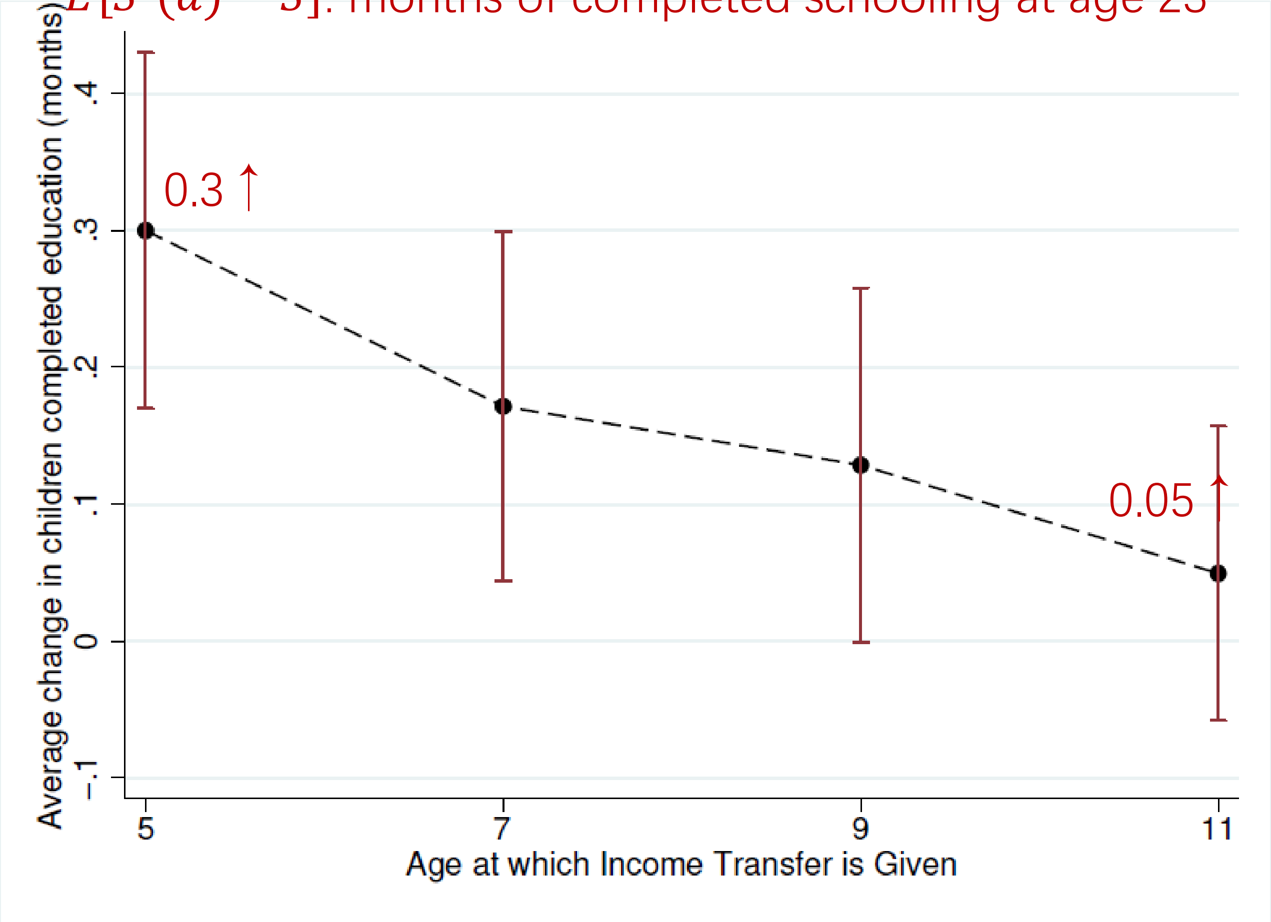


Figure 8: Average Effect of an Income Transfer by Age of Transfer (Outcome: Schooling at Age 23)

$E[S'(a) - S]$: months of completed schooling at age 23



3. Policy Experiments

Pre:王祎

- Estimate this heterogeneity in the policy treatment effects by child's initial skills (Y0) and initial family income levels (θ0)

Figure 9: Heterogeneity in Policy Effects by Age 5-6 Household Income (Outcome: Schooling at Age 23)

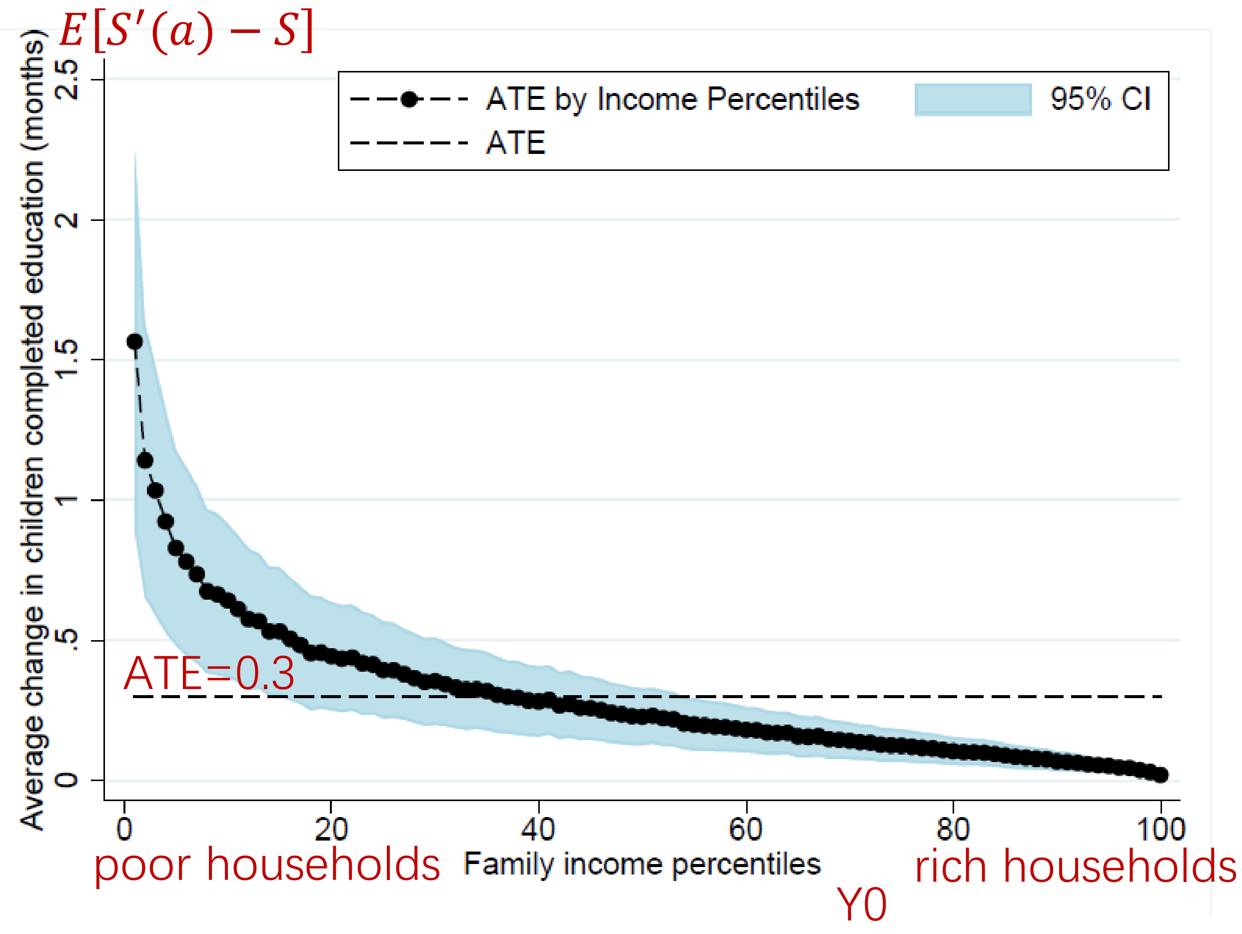
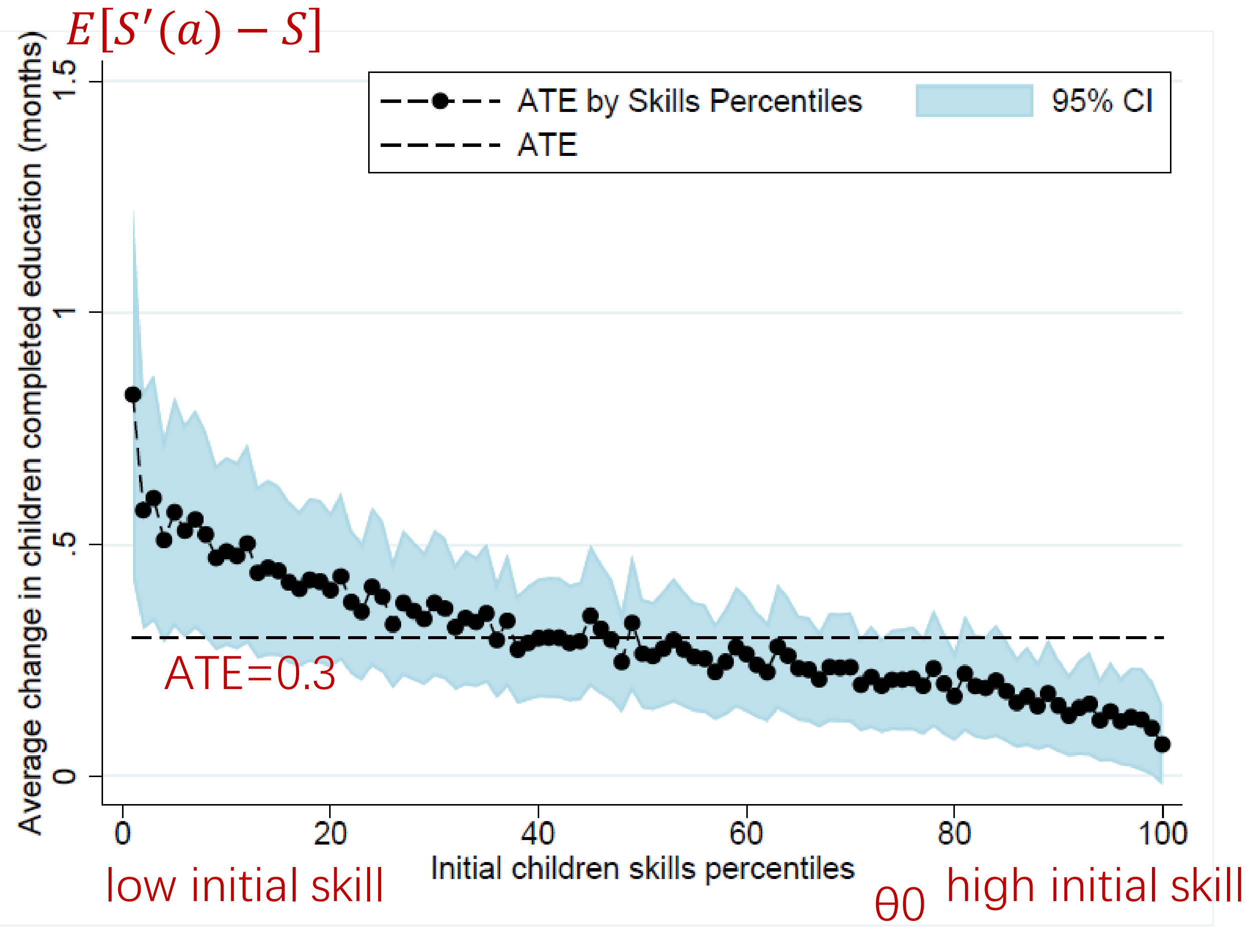


Figure 10: Heterogeneity in Policy Effects by Age 5-6 Children's Skills (Outcome: Schooling at Age 23)



4. Quantifying the Importance of Measurement Error

Pre:王祎

- Discuss how the estimates of the primitive production technology would differ if we ignore the measurement error issues

Table 3: Estimated Policy Effects and Measurement Error

Panel A: ATE by Age of Income Transfer

Model	Age of Income Transfer ($ 10,000) Age 5-6	Age 7-8	Age 9-10	Age 11-12
Measur Error Corrected	0.300 [0.170,0.430]	0.172 [0.044,0.299]	0.129 [-0.001,0.258]	0.050 [-0.058,0.157]
Not Corrected for Measur Error	0.062 [0.042,0.082]	0.029 [0.014,0.044]	0.022 [0.007,0.036]	0.013 [-0.000,0.027]

Panel B: ATE at age 5-6 by Family Income

	Low Income Families (10th Income Percentile)	High Income Families (90th Income Percentile)
Measur Error Corrected	0.642 [0.369,0.915]	0.069 [0.034,0.105]
Not Corrected for Measur Error	0.126 [0.085,0.166]	0.018 [0.012,0.023]

A substantial reduction in the estimated effect of an income transfer.

5. Cost-Benefit Analysis

Pre:王祎

- Providing income transfers to families would produce positive gains in children's skills, with larger effects for poorer households.
- Present a simple cost-benefit analysis that focuses on an income transfer policy to the poor families (10th percentile of Y0)

Table 4: Average Effect of an Income Transfer by Age of Transfer (Outcome: PV of Earnings)

Age of Intervention	Benefit on PV Earnings ($)	Direct Cost (Income Transfer) ($)	Cost of Education ($)	Net Benefit ($)
5	29126.81	10000	749.58	18377.23
7	15598.78	10000	403.65	5195.14
9	11541.43	10000	299.12	1242.31
11	4365	10000	113.43	-5748.43

Pre:黄涛

6 Conclusion

- In this paper, a new recognition result is developed based on the measures of skill measurement restriction.
- Based on the recognition results, a robust sequential estimation algorithm is developed.
- This paper uses American data and a flexible skill development parameter model to estimate the skill production process, which allows the complementarity of free skill production between children's skill stock and parents' investment.
- Parameter estimation shows that: Investment is more effective in the early stage, especially for disadvantaged children.
- The research results show that: Income transfer has a positive return at an early age, especially for poorer families
- The research results show that: For children's skills, family income is a better "goal" than initial children's skills.
- Finally, we find that the policy impact of estimation is attenuated by the bias of measurement error, which proves that it is crucial to correct the estimation measurement error.

Thanks